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A MATHEMATICAL MODEL OF AIRCRAFT FOR THE INVESTIGATION OF NONSTATIONARY AERODYNAMIC CHARACTERISTICS

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Basic proportions are defined for setting a computation scheme for the determination of aerodynamic characteristics of an aircraft on a computer. A schematic representation of an aircraft by a system of reference elements is described and substantiated. The general nonstationary linear problem of aerodynamics and hydrodynamics is reduced to a set of canonical ε -problems that are solved independently. General properties of linear characteristics (loads, normal forces, and moments) are established. Integral relationships of the convolution kind which yield explicit formulas for these characteristics in terms of related transition functions and laws of kinematic parameter variations with time $\varepsilon_j(\tau)$ are derived. A number of general theorems, including the generalization of the invertibility theorem, are proved. Exact formulas are established for transition functions at the initial instant of time.

1. The general nonstationary linear problem. Let us consider the unsteady motion and the deformations of an aircraft in a perturbed continuous medium. We attach to the aircraft a conventional Cartesian system of coordinates whose Ox-axis is directed forward along the aircraft axis and the Oz-axis directed to the right over the wing span (Fig. 1). Let U_0 be the mean velocity of the (coordinate) origin 0, and W, W^*, W_Δ and W_δ be the velocity vectors of perturbations induced by the aircraft, transfer, gusts of the medium, and aircraft surface deformations, respectively. We denote the vector of absolute angular velocity by Ω and time by t ($t = 0$ is the instant of unstable processes onset), and introduce the dimensionless quantities

$$\xi = \frac{x}{b}, \quad \eta = \frac{y}{b}, \quad \zeta = \frac{z}{b}, \quad \tau = \frac{u_0 t}{b}$$

$$\omega_{x, y, z}(\tau) = \frac{b\Omega_{x, y, z}}{U_0}, \qquad w_{x, y, z\Delta}(\xi, \eta, \zeta, \tau) = \frac{W_{x, y, z\Delta}}{U_0}$$

where b is a characteristic linear dimension.

Let uU_0 be the varying velocity component of origin O, and α and β the angles of attack and of the side slip, respectively. For projections of the transfer velocity at any point of the aircraft surface we then have

$$\frac{W_x^*}{U_0} = (1+u)\cos\alpha\cos\beta + \omega_y\zeta - \omega_z\eta,\ \frac{W_y^*}{U_0} = -(1+u)\sin\alpha\cos\beta - \omega_x\zeta + \omega_z\xi,\ \frac{W_z^*}{U_0} = (1+u)\sin\beta - \omega_x\eta - \omega_y\xi$$

The aircraft motion, deformations of its surface and gust velocities of the medium are assumed to be known. Values of perturbed parameters of gas, in the first instance the aerodynamic loads, are to be determined. Usually these can be determined on the assumption of a perfect medium. Condition of impenetrability apply to the aircraft surface. It is of the form

$$W_n / U_0 = W_n^* / U_0 + W_{\delta n} / U_0 - W_{\Delta n} / U_0 \qquad (1.1)$$

where n is the unit vector of an outward normal to the surface of the body.

Conditions at infinity, at discontinuity surfaces, at the free sheet, at separation boundaries, and the hypothesis of Chaplygin-Zhukovskii must be satisfied.

Below, the general nonstationary linear problem is considered, in which the perturbations induced by the aircraft are assumed small in comparison with parameters of the unperturbed medium. The flow around the aircraft is conveniently defined with the use of gasdynamic properties that satisfy the equation of continuity outside the aircraft and are applied at reference planes. The latter are located closely to the related part of the aircraft surface. The basic scheme of an aircraft is shown in Fig. 1. The plane reference elements, which can be parallel to the Ox-axis, are shown there by solid lines. Modifications of this scheme are possible; for instance, additional reference planes must be introduced for detailed investigation of flow near the wing-fuselage joint. We write the equation of a plane reference element i as

$$a_i + \eta\cos(ny)_0^i + \zeta\cos(nz)_0^i = 0$$

By the linearization condition from (1.1) we have at the surface of the aircraft

$$|W_n^* / U_0| \ll 1, \quad |W_{\delta n} / U_0| \ll 1, \quad |W_{\Delta n} / U_0| \ll 1$$

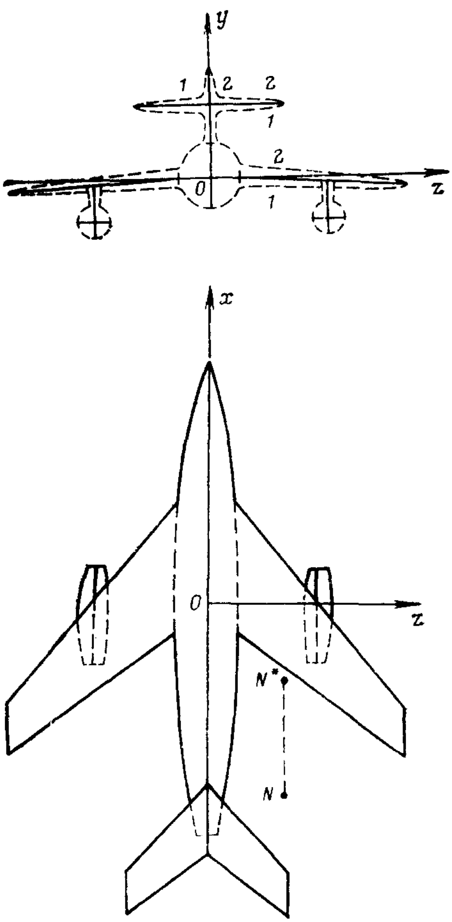

Fig. 1

from which we obtain the most important case

$$|\cos(nx)| \ll 1, \quad \left|\frac{W_{y,z}^*}{U_0}\right| \ll 1, \quad \left|\frac{W_{x,y,z\delta}}{U_0}\right| \ll 1, \quad \left|\frac{W_{x,y,z\Delta}}{U_0}\right| \ll 1$$

Let 1 and 2 denote different sides of each reference element, with the positive normal

$n_0{}^i$ directed from side 1 to side 2. We write the equations for the i-th aircraft part and its deformation (including deflections of control surfaces) in the form

$$\frac{(\Delta n_0{}^i)_{1,2}}{b} = \sum_\nu \delta_\nu{}^i(\tau) f^i_{\delta_\nu 1,2}(\xi, \eta, \zeta) \tag{1.2}$$

The thickness of aircraft parts is defined by the sum of deformations at different sides of the reference plane, and their curvature by the half-difference of these. We introduce the following functions of the aircraft thickness and curvature

$$f^{*i}_{\delta_\nu} = f^i_{\delta_\nu 1} + f^i_{\delta_\nu 2}, \quad 2f^i_{\delta_\nu} = f^i_{\delta_\nu 2} - f^i_{\delta_\nu 1}$$

The dimensionless velocities of gusts are expressed in a similar form

$$w_{x,y,z\Delta} = \sum_k \Delta^k_{x,y,z}(\tau)\, w^k_{x,y,z\Delta}(\xi, \eta, \zeta) \tag{1.3}$$

In linear problems the velocity field disturbed by a body can be considered to be potential. Within acceptable accuracy shocks coincide with Mach lines whose position relative to the body remain unchanged. The vortex trail running off the aircraft surface can be positioned in the related reference elements, and the boundary condition can be related to the nearest side of the reference plane. All this makes it possible to reduce the general problem to a number of particular conventional ones that can be solved independently of each other.

We denote by $\varepsilon_j, j = 1, 2, 3, \ldots$ the set of basic dimensionless kinematic parameters that define the motion and deformation of the aircraft and the gusts of the medium. If $\varepsilon_j{}^*$ denotes their characteristic values, the dimensionless potential and aerodynamic loads on the element i can be represented by

$$\varphi(\xi, \eta, \zeta, \tau) = \sum_j \varepsilon_j{}^* \varphi_{\varepsilon_j}(\xi, \eta, \zeta, \tau), \quad \Delta p^i = p_1{}^i - p_2{}^i \tag{1.4}$$

$$\Delta p^i(\xi, \eta, \zeta, \tau) = \sum_j \varepsilon_j{}^* \Delta p^i_{\varepsilon_j}(\xi, \eta, \zeta, \tau), \quad \Delta p^i_{\varepsilon_j} = p^i_{\varepsilon_{j1}} - p^i_{\varepsilon_{j2}}$$

where φ_{ε_j} and $\Delta p_{\varepsilon_j}{}^i$ are solutions of the particular problem in which all kinematic parameters, except ε_j, are zero. Each function φ_{ε_j} satisfies outside the aircraft the same equation as φ (a_0 is the speed of sound in the unperturbed gas)

$$(1 - M^2)\frac{\partial^2 \varphi_{\varepsilon_j}}{\partial \xi^2} + \frac{\partial^2 \varphi_{\varepsilon_j}}{\partial \eta^2} + \frac{\partial^2 \varphi_{\varepsilon_j}}{\partial \zeta^2} + 2M\frac{\partial^2 \varphi_{\varepsilon_j}}{\partial \xi\, \partial \tau} - M^2 \frac{\partial^2 \varphi_{\varepsilon_j}}{\partial \tau^2} = 0, \quad M = \frac{U_0}{a_0}$$

Conditions at the vortex sheet are conveniently written in the form [1]

$$\Delta\varphi_{\varepsilon_j}(\xi, \eta, \zeta, \tau) = \Delta\varphi_{\varepsilon_j}(\xi^*, \eta, \zeta, \tau^*), \quad \Delta\varphi_{\varepsilon_j} = \varphi_{\varepsilon_j 1} - \varphi_{\varepsilon_j 2}, \quad \tau^* = \tau - (\xi^* - \xi)$$

It links the potential difference on the opposite sides of the vortex sheet at two points: immediately behind the reference plane $N^*(\xi^*, \eta, \zeta)$ and downstream of it at any point $N(\xi, \eta, \zeta)$ with the same values of η and ξ (Fig. 1).

To satisfy the Chaplygin-Zhukovskii condition at the trailing edges of reference planes it is necessary to stipulate continuity of velocities $\partial\varphi_{\varepsilon_j} / \partial\xi$, $\partial\varphi_{\varepsilon_j} / \partial\eta$ and $\partial\varphi_{\varepsilon_j} / \partial\zeta$. in the vicinity of these. For $M > 1$ and subsonic velocities at trailing edges, it is sufficient if functions φ_{ε_j} themselves are continuous [1].

The dimensionless loads are determined by the Cauchy-Lagrange integral which in the linear form is written as

$$\Delta p^i_{\varepsilon_j} = 2\,(\partial\Delta\varphi_{\varepsilon_j} / \partial\xi - \partial\Delta\varphi_{\varepsilon_j} / \partial\tau),$$

Conditions at infinity reduce to the requirement that the velocities and pressures distributed by the aircraft, as well as functions φ_{ε_j}, vanish far ahead of the latter (for $M > 1$, beyond the influence region).

Boundary conditions for any point (ξ, η, ζ), belonging to a reference element can be represented in the form

$$\frac{\partial\varphi_{\varepsilon_j}}{\partial n_0{}^i} = \frac{\varepsilon_j(\tau)}{\varepsilon_j{}^*} F^i_{\varepsilon_j}(\xi, \eta, \zeta), \qquad \frac{\partial\varphi_{\varepsilon_j}}{\partial n_0{}^i} = \frac{\partial\varphi_{\varepsilon_j}}{\partial\eta}\cos{(ny)_0}^i + \frac{\partial\varphi_{\varepsilon_j}}{\partial\zeta}\cos{(nz)_0}^i \tag{1.5}$$

where $F_{\varepsilon_j}{}^i(\xi, \eta, \zeta)$ are known functions expressed in terms of the aircraft geometry and of specified functions appearing in (1.2) and (1.3)

$$F_u{}^i = 0,\ F_\alpha{}^i = \cos{(ny)_0}^i,\ F_\beta{}^i = \cos{(nz)_0}^i,\ F^i_{\omega_x} = \eta\cos{(nz)_0}^i -$$
$$\zeta\cos{(ny)_0}^i,\ F^i_{\omega_y} = -\xi\cos{(nz)_0}^i$$
$$F^i_{\omega_z} = \xi\cos{(ny)_0}^i,\ F^i_{\Delta_x^k} = 0,\ F^i_{\Delta_y^k} = -w^k_{y\Delta}\cos{(ny)_0}^i,\ F^i_{\Delta_z^k} = -w^k_{z\Delta}\cos{(nz)_0}^i \ldots$$

Since the boundary condition (1.1) contains deformation rates, hence the set of basic kinematic parameters ε_j includes also derivatives with respect to $\tau\,\delta_\nu{}^{\cdot i}(\tau)$

$$\varepsilon_j(\tau) \in \{\alpha(\tau),\ \beta(\tau),\ \omega_{x,y,z}(\tau),\ \Delta_y{}^k(\tau),\ \Delta_z{}^k(\tau),\ \delta_\nu{}^i(\tau),\ \ \delta_\nu{}^{\cdot i}(\tau)\}$$

The conditions of linearization of the problem are of the form

$$|\cos(nx)| \ll 1, \quad |\varepsilon_j(\tau)| \ll 1, \quad |u(\tau)| \ll 1$$

2. Integral representation. Let us consider the somewhat particular canonical problem of the so-called transition function of the potential $[\varphi_{\varepsilon_j}(\xi, \eta, \zeta, \tau)]$. Its conditions differ from those indicated above only by another dependence of the boundary condition on time

$$\frac{\partial[\varphi_{\varepsilon_j}]}{\partial n_0{}^i} = 1(\tau)\,F^i_{\varepsilon_j}(\xi, \eta, \zeta), \qquad 1(\tau) = \begin{cases} 0, & \tau < 0 \\ 1, & \tau \geqslant 0 \end{cases} \tag{2.1}$$

We introduce in the analysis the nonstationary part of the transition function, separating the limit value for $\tau \to \infty$. For the aerodynamic load let us, for example, set

$$I_{pi}{}^{\varepsilon_j}(\xi, \eta, \zeta, \tau) = [\Delta p_{\varepsilon_j}{}^i(\xi, \eta, \zeta, \tau)] - \Delta p_i{}^{\varepsilon_j}(\xi, \eta, \zeta) \tag{2.2}$$

where $\Delta p_i{}^{\varepsilon_j}(\xi, \eta, \zeta)$ is a stationary aerodynamic derivative.

Using Duhamel's integral, as was done in [1] for a wing, we can readily obtain the general integral formula for the load. Let for $\tau < 0$ the kinematic parameter $\varepsilon_j(\tau) = 0$, and for $\tau = 0$ jumps to $\varepsilon_j(0)$, and after that varies continuously. Then

$$\Delta p^i_{\varepsilon_j}(\xi, \eta, \zeta, \tau) = \Delta p_i^{\varepsilon_j}(\xi, \eta, \zeta)\,\varepsilon_j(\tau) + I_{pi}^{\varepsilon_j}(\xi, \eta, \zeta, \tau)\,\varepsilon_j(0) + \tag{2.3}$$
$$\mu\Delta^* p_i^{\varepsilon_j^{\cdot}}(\xi, \eta, \zeta)\,\varepsilon_j^{\cdot}(\tau) + \int_0^\tau \varepsilon_j^{\cdot}(\tau - \tau_1)\,I_{pi}^{\varepsilon_j}(\xi, \eta, \zeta, \tau_1)\,d\tau_1, \quad \mu = \begin{cases} 0, & M > 0 \\ 1, & M = 0 \end{cases}$$

The first term in the right-hand part of (2.3) is the mathematical expression of the stationarity hypothesis, while the second represents the correction due to the nonstationarity of flow around a body. The integral in the right-hand part of (2.3) together with the one but last term provides the explicit expression of the effect of nonstationary flow for continuous variation of the kinematic parameter. In a compressible medium ($M > 0$) when the propagation velocity of perturbations is finite, the third term vanishes. In an incompressible fluid ($M = 0$) the speed of sound is infinite, and the variation of $\varepsilon_j(\tau)$ is accompanied by the emergence of pulses of the delta-function kind. The third term appeared as the consequence of isolation of the indicated singularity in the integrand. The load $\Delta^* p\varepsilon_j$ is obtained by solving the ε_j-problem for a noncirculation flow past the aircraft.

Integral representations similar to (2.3) are also valid for other linear aerodynamic characteristics. Formulas (2.3) together with (1.4) yield explicit expressions of (the dependence of) the indicated characteristics on the previous history of flow (on the laws of kinematic parameter variation with time). It is important that in applications to flight dynamics, aeroelasticity, etc. it is not necessary to know the laws of $\varepsilon_j(\tau)$ in order to determine $\Delta p_i^{\varepsilon j}$, $\Delta^* p_i^{\varepsilon j}$, $I_{pi}^{\varepsilon j}$ and similar functions [2, 3].

If the kinematic parameters vary according to the harmonic law $\varepsilon_j(\tau) = \varepsilon_j^* \cos(p_j^*\tau + \theta_j)$, then the loads and other linear characteristics are expressed in terms of aerodynamic derivatives independent of time

$$\Delta p_i(\xi, \eta, \zeta, \tau) = \sum_j \{\Delta p_i^{\varepsilon j}(\xi, \eta, \zeta, p_j^*)\,\varepsilon_j(\tau) + \Delta p_i^{\varepsilon j^{\cdot}}(\xi, \eta, \zeta, p_j^*)\,\varepsilon_j^{\cdot}(\tau)\}$$

The integral formulas are in that case transformed to

$$\Delta p_i^{\varepsilon j}(\xi, \eta, \zeta, p_j^*) = \Delta p_i^{\varepsilon j}(\xi, \eta, \zeta) + p_j^* \int_0^{\tau_j^*} I_{pi}^{\varepsilon j}(\xi, \eta, \zeta, \tau) \sin p_j^*\tau\, d\tau \tag{2.4}$$

$$\Delta p_i^{\varepsilon j^{\cdot}}(\xi, \eta, \zeta, p_j^*) = \mu\Delta^* p_i^{\varepsilon j^{\cdot}}(\xi, \eta, \zeta) + \int_0^{\tau_j^*} I_{pi}^{\varepsilon j}(\xi, \eta, \zeta, \tau) \cos p_j^*\tau\, d\tau$$

$$I_{pi}^{\varepsilon j}(\xi, \eta, \zeta, \tau) = 0, \quad \tau > \tau_j^*$$

At subsonic velocities at which the transition process is of an asymptotic character $\tau_j^* = \infty$.

The derived formulas permit a strict closure of linear equations of flight dynamics and aeroelasticity [2, 3] and materially reduce the volume of theoretical and experimental investigations in aerodynamics [1].

3. Exact solutions. Let us consider a stepwise variation of the boundary condition with respect to time (2.3). At the initial instant of the transition process ($\tau > 0$, $\tau \to 0$) each element of the aircraft surface generates perturbations independently of one another, owing to the lag in the propagation of these in a compressible medium. Hence all assumptions on which the piston theory is based are strictly satisfied, and that theory yields exact values for transition functions when $\tau \to 0$ [1].

Let $w_i(\xi, \eta, \zeta)$ be the velocity imparted pulse-like to the gas at $\tau = 0$ at any point (ξ, η, ζ) of the reference plane i. The related perturbed pressure is then defined by formula [1]

$$[p_i'(\xi, \eta, \zeta, 0)] = \pm q_\infty \frac{2}{M} \frac{w_i(\xi, \eta, \zeta)}{U_0}, \qquad q_\infty = \frac{\rho_\infty U_0^2}{2}$$

where the sign depends on whether a compression (the plus sign) or rarefaction (the minus sign) flow is considered. For the chosen direction of the normal we can write

$$[p_i'(\xi, \eta, \zeta, 0)]_{1,2} = \mp q_\infty \frac{2}{M} \frac{\partial \psi}{\partial n_0^i}$$

with allowance for (1.4) and (1.5) from this we obtain

$$[\Delta p'_{\varepsilon_j}(\xi, \eta, \zeta, 0)] = -\frac{4}{M} F^i_{\varepsilon_j}(\xi, \eta, \zeta)$$

Integrating these formulas, we readily obtain initial values of transition functions for the over-all characteristics.

It will be seen that the second limit value of the transition function (for $\tau \to \infty$) is the same as the stationary value of the related aerodynamic derivative

$$[\Delta p^i_{\varepsilon_j}(\xi, \eta, \zeta, \infty)] = \Delta p_i^{\varepsilon_j}(\xi, \eta, \zeta), \qquad \Delta p_i^{\varepsilon_j}(\xi, \eta, \zeta) = \Delta p_i^{\varepsilon_j}(\xi, \eta, \zeta, p_j^*)\Big|_{p_j^*=0}$$

4. General theorems. General theorems help to establish a rational computation scheme for a systematic application of numerical methods, and provide important means for checking the correctness of computations and the exactness of results. Below we present a number of theorems that are important from that point of view.

The integral formulas (2.3) and (2.4) are used for proving Theorems 1 — 4.

Theorem 1. The momentum. The momentum of the nonstationary part of the transition function of any linear characteristic throughout the time of a transition process in a compressible medium is equal to the related aerodynamic derivative with a dot for $p_j^* \to 0$. In an incompressible medium it is equal to the difference of such derivatives for circulation and circulationless flows past a body.

For loads we have [1, 4]

$$\int_0^{\tau_j^*} I_{pi}^{\varepsilon_j}(\xi, \eta, \zeta, \tau)\, d\tau = p_i^{\varepsilon_j^{\cdot}}(\xi, \eta, \zeta, 0) - \mu\Delta^* p_i^{\varepsilon_j^{\cdot}}(\xi, \eta, \zeta) \tag{4.1}$$

$$I_{pi}^{\varepsilon_j}(\xi, \eta, \zeta, \tau) = 0, \qquad \tau > \tau_j^*$$

Note. Strictly speaking, the theorem is proved on the assumption that the momentum is finite. For wings of infinite span and subsonic velocities integrals (4.1) are usually divergent. Thus in an incompressible medium, when exact solutions for a slender profile exist, the nonstationary part od the transition function for considerable τ decreases as $1/\tau$. However in all these cases the aerodynamic derivatives with a dot also become infinite for $p_j^* \to 0$. Hence equality (4.1) is also satisfied here.

Theorem 2. The aerodynamic derivative without a dot. For very high values of p_j^* the aerodynamic derivatives with respect to ε_j tend to values that correspond to transition functions at the instant of the transition process onset. For an incompressible medium the continuous part of the transition function is to be used.

For aerodynamic loads the theorem is of the form

$$\Delta p_i^{\varepsilon_j}(\xi, \eta, \zeta, \infty) = [\Delta p^i_{\varepsilon_j}(\xi, \eta, \zeta, 0)]$$

Theorem 3. The aerodynamic derivative with a dot. For very considerable p^* the aerodynamic derivatives with respect to $\varepsilon_j^{\cdot}$ tend to values of the circulation-free derivatives. For a compressible medium the indicated limit values are zero.

For aerodynamic loads the theorem is of the form

$$\Delta p_i^{\varepsilon_j}(\xi, \eta, \zeta, \infty) = \mu\Delta^* p_i^{\dot{\varepsilon}_j}(\xi, \eta, \zeta)$$

Theorem 4. Mean value. If variation of the kinematic parameter is periodic of period τ_T, the mean integral value of any linear nonstationary characteristic during period τ_T is equal to the product of the stationary derivative by the mean integral value of the kinematic parameter during that period.

For continuously varying $\varepsilon_j(\tau)$ the theorem for aerodynamic load is of the form

$$\frac{1}{\tau_T}\int\limits_0^{\tau_T}\Delta p^i_{\varepsilon_j}(\xi,\eta,\zeta,\tau)\,d\tau=\Delta p_i^{\varepsilon_j}(\xi,\eta,\zeta)\frac{1}{\tau_T}\int\limits_0^{\tau_T}\varepsilon_j(\tau)\,d\tau$$

The theorem is readily extended to the case in which function $\varepsilon_j(\tau)$ has a finite number of discontinuities of the first kind during one period.

Theorem 5. Reversibility. This theorem establishes an integral relationship between boundary conditions and the corresponding to these aerodynamic loads on an aircraft in direct or inverted motion with respect to velocity U_0. The proposed aircraft schematization and the related formulation of the problem described in Sect. 1 makes it possible to extend the theorem on reversibility to the considered case, as was done in [1].

We denote by $\varphi^*(\xi,\eta,\zeta,\tau)$ that part of the velocity potential whose normal velocities continuously vary at passage through the reference planes. Then for any arbitrary functions $\varepsilon_j(\tau)$ we have

$$\sum_i\iint\limits_{\sigma_i}\Delta p_{i+}\frac{\partial\varphi_-^*}{\partial n_0{}^i}\,d\sigma_i=\sum_i\iint\limits_{\sigma_i}\Delta p_{i-}\frac{\partial\varphi_+^*}{\partial n_0{}^i}\,d\sigma_i \tag{4.2}$$

where σ_i is the dimensionless area of the reference element i, and the subscripts plus and minus relate to the direct and inverted motions, respectively.

5. Corollaries of the reversibility theorem. Equality (4.2) yields important formulas for over-all characteristics. We shall use conventional attached systems of coordinates for both direct and inverted motions in which

$$\xi_+=-\xi_-,\quad \eta_+=\eta_-,\quad \zeta_+=-\zeta_-,\quad \cos(ny)_{0+}{}^i=\cos(ny)_{0-}{}^i,\quad \cos(nz)_{0+}{}^i=-\cos(nz)_{0-}{}^i$$

We introduce coefficients of the normal and side forces and moments relative to the attached axes

$$c_{y,z}=\frac{F_{y,z}}{q_\infty s},\qquad m_{x,y,z}=\frac{M_{x,y,z}}{q_\infty sb}$$

where s is a characteristic area.

Using formula (4.2) with allowance for (1.4) – (1.6), we establish the relation between the characteristics of a rigid aircraft in direct and inverted motions. If the kinematic parameters vary according to any arbitrary law, then

$$c_{y\alpha+}(\tau)=c_{y\alpha-}(\tau),\qquad c_{z\beta+}(\tau)=c_{z\beta-}(\tau) \tag{5.1}$$

$$m_{x\omega_x+}(\tau)=m_{x\omega_x-}(\tau),\quad m_{y\omega_y+}(\tau)=m_{y\omega_y-}(\tau),\quad m_{z\omega_z+}(\tau)=m_{z\omega_z-}(\tau)$$

If the laws of variation of the considered kinematic parameters for direct and inverted motions are the same, i.e. $\varepsilon_{j+}(\tau)=\varepsilon_{k-}(\tau)$, we have the following equalities:

$$m_{x\alpha+}(\tau)=c_{y\omega_x-}(\tau),\ m_{x\beta+}(\tau)=c_{z\omega_x-}(\tau),\ m_{y\alpha+}(\tau)=-c_{y\omega_y-}(\tau) \tag{5.2}$$

$$m_{y\beta-}(\tau)=-c_{z\omega_y-}(\tau),\ m_{y\omega_x-}(\tau)=-m_{x\omega_y-}(\tau),\ m_{z\alpha+}(\tau)=c_{y\omega_z-}(\tau)$$

$$m_{z\beta+}(\tau)=c_{z\omega_z-}(\tau),\ m_{z\omega_x+}(\tau)=m_{x\omega_z-}(\tau),\ c_{z\alpha+}(\tau)=c_{y\beta-}(\tau)$$

$$m_{z\omega_y+}(\tau) = -m_{y\omega_z-}(\tau)$$

For an aircraft symmetric with respect to the Oxy-plane some of these coefficients are zero. The equalities (5.1) and (5.2) are valid when the subscripts plus and minus are interchanged. Hence all linear over-all coefficients for a rigid aircraft moving in a straight line in a quiescent medium are defined by characteristics of inverse motion for the same number M. Similar equalities hold also for the coefficients of aerodynamic derivatives.

The reversibility theorem makes possible a simplification of computation of forces and moments induced by changes of curvature of each aircraft part i, since from (4.2) it is possible to obtain [1]

$$c^i_{y\delta_\nu+} = \iint\limits_{\sigma_i} \Delta p^i_{\alpha-} \left(\frac{\partial f^i_{\delta_\nu}}{\partial \xi}\right)_+ d\sigma_i, \quad c^i_{y\delta_\nu\cdot+} = -\iint\limits_{\sigma_i} \Delta p^i_{\alpha-} f^i_{\delta_\nu+} d\sigma_i \tag{5.3}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$m^i_{z\delta_\nu+} = \iint\limits_{\sigma_i} \Delta p^i_{\omega_z-} \left(\frac{\partial f^i_{\delta_\nu}}{\partial \xi}\right)_+ d\sigma_i, \quad m^i_{z\delta_\nu\cdot+} = -\iint\limits_{\sigma_i} \Delta p^i_{\omega_z-} f_{\delta_\nu+} d\sigma_i$$

It will be seen from (5.3) that for determining any of the indicated characteristics it is not necessary to consider each time a new problem of aerodynamics. It is sufficient to solve the α-, β- and $\omega_{x,y,z}$-problems in inverse motion and thus reduce the considered problem to the computation of integrals.

The proposed aircraft schematization has proved to be highly effective in practice by allowing to comparatively simply generalize the numerical methods originally developed for wings [1]. It proves fairly economic, and in several instances suitable for using on a computer of average capacity. Systematic computations based on this method are in satisfactory agreement with related experimental data throughout the existing range of actual flight numbers M, as long as the linear dependence of aerodynamic characteristics on ε_j are maintained. The presented above exact formulas and theorems have helped to create a reliable control system for numerical calculations on a computer and to reduce their volume.

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